

# COLOR AND POPULATION GRADIENTS IN THE CORE OF THE POSTCOLLAPSE GLOBULAR CLUSTER M30<sup>a)</sup>

GIAMPAOLO PIOTTO

Astronomy Department, University of California, Berkeley, California 94720

and

Dipartimento di Astronomia, Università di Padova, Padova, Italy

IVAN R. KING<sup>b)</sup>

Astronomy Department, University of California, Berkeley, California 94720

S. DJORGOVSKI<sup>b)</sup>

Division of Physics, Mathematics, and Astronomy, California Institute of Technology, Pasadena, California 91125

Received 14 June 1988; revised 25 August 1988

## ABSTRACT

We have developed a new technique for seeking color gradients in multicolor CCD images of a globular cluster. Application of this technique to M30 confirms the radial color gradient reported by others. The  $B - V$  and  $V - R$  colors show linear gradients, when plotted against  $\log r$ , that are many times their statistical errors. In the radial range  $\sim 3$ –100 arcsec, the  $B - V$  and  $V - R$  colors increase outwards by about  $(0.18 \pm 0.015)$  mag/dex and  $(0.09 \pm 0.01)$  mag/dex, respectively. Color-magnitude arrays of the central region show that the Balmer-line strength that accompanies the central blueness is due to an increased proportion of blue-horizontal-branch starlight in the center. This excess does not appear to be statistically significant in terms of star numbers, even though the color gradient is so highly significant. The authors are divided among themselves as to whether the color and spectrum gradients observed here and in other clusters are due only to random statistical fluctuations, or whether they might have a physical significance.

## I. INTRODUCTION

The tendency towards equipartition in stellar systems should induce a mass segregation. Globular clusters have long been recognized as good candidates for such processes to occur, and, in particular, as a consequence of formation of close binaries during core collapse (e.g., Lee 1987). However, the existence of mass segregation has not been unambiguously established in any real globular cluster until recently (see the interesting papers on blue stragglers by Nemec and Harris (1987) and by Nemec and Cohen (1988), and the forthcoming paper on faint stars in M71 by Richer and Fahlman (1988)).

One observable manifestation of a mass segregation may be the existence of a color gradient, and several groups in the past have searched for one. Chun and Freeman (1979) claimed the detection of gradients in several clusters, based on photoelectric aperture photometry, in the sense that the centers were redder. This result is now widely regarded as spurious, and caused by a bias in centering of the smallest apertures used in the photometry: one tends to pick a chance lump of red giants. Scaria and Bappu (1981) claimed a color variation within  $\omega$  Cen, which has not been confirmed. Peterson (1986) compiled photoelectric aperture photometry for 101 clusters, and found that some showed color gradients in the sense of being bluer towards the center, and some in the opposite sense; given the low accuracy of the data, he concluded that these apparent gradients may be caused by statistical fluctuations in the number of bright stars. Hanes and Brodie (1985), on the other hand, found no significant

color gradients in their photoelectric aperture-photometry study of 71 Galactic globulars. On a more subtle level, Buonanno *et al.* (1986) claimed that the fainter and hotter blue-horizontal-branch stars are distributed differently from their cooler and brighter counterparts, whereas Bailyn *et al.* (1988a) found no significant color gradients in six x-ray globular clusters. However, in their higher-resolution data on M15, Bailyn *et al.* (1988b) show a highly significant color gradient, in the sense of being bluer toward the center. Meylan (1982) found an increase with radius in the number ratio of red and blue supergiants in nine young globulars in the LMC, and attributed the effect to delayed star formation in the clusters' cores.

In two cases, M30 (C2137 – 234 = NGC 7099) and NGC 4147 (C1207 + 188), spectra also show a strengthening of the Balmer lines in the cluster cores (Hesser and Shawl 1985, and references therein; Rose, Stetson, and Trippico 1987, hereafter referred to as RST), suggesting the presence of some enhanced population of hot stars. This Balmer-line gradient was also noted by Zinn and West (1984), who speculated on the origin of this anomaly, and its possible connection with the color gradients suggested by other authors. Friel, Heasley, and Christian (1987) mention that the central region of NGC 4147 appears to be bluer than the outer regions of that cluster. One of our purposes here is to identify the source of the spectroscopic anomaly in M30.

## II. COLOR GRADIENTS

Williams and Bahcall (1979), Chun and Freeman (1979), Cordoni and Aurière (1984), and Peterson (1986) have all noticed a color gradient in the center of M30, in the sense of being bluer towards the center. Djorgovski and Penner (1985) noticed that power law fits to the surface brightness in the central cusp of M30 give different results depending on whether the UV photographic data or the red CCD

<sup>a)</sup> Based on observations collected at the European Southern Observatory, La Silla, and the Cerro Tololo Inter-American Observatory.

<sup>b)</sup> Visiting Astronomer, Cerro Tololo Inter-American Observatory, which is operated by the Association of Universities for Research in Astronomy, Inc., under contract with the National Science Foundation.

data were used; but nothing was made of this result, because of a multitude of possible systematic errors in comparing data obtained in different conditions, with different detectors, and processed in a slightly different way. A subsequent examination of a homogeneous CCD surface-photometry dataset obtained at CTIO for a survey of postcollapse cores also indicated that the cusp slope is steeper in the bluer bandpasses. Prompted by this observation, and the results of RST, we undertook a more detailed study of M30, and to a lesser extent NGC 4147. In this paper we report on the results on M30 only, and postpone the study of NGC 4147 and other clusters for a future publication.

We carried out surface photometry on two sets of CCD frames of M30, both consisting of  $B$ ,  $V$ , and  $R$  exposures. One set was taken with a GEC  $576 \times 385$  chip on the 1.5 m reflector of Cerro Tololo Inter-American Observatory, the other with an RCA  $512 \times 320$  chip on the 2.2 m reflector of the European Southern Observatory. The seeing FWHM was  $\simeq 2.4$ – $2.5$  arcsec for the CTIO data, and  $\simeq 1.3$ – $1.5$  arcsec for the ESO data, with pixel scales of 0.398 and 0.365 arcsec, respectively. The frames were reduced using the standard procedures.

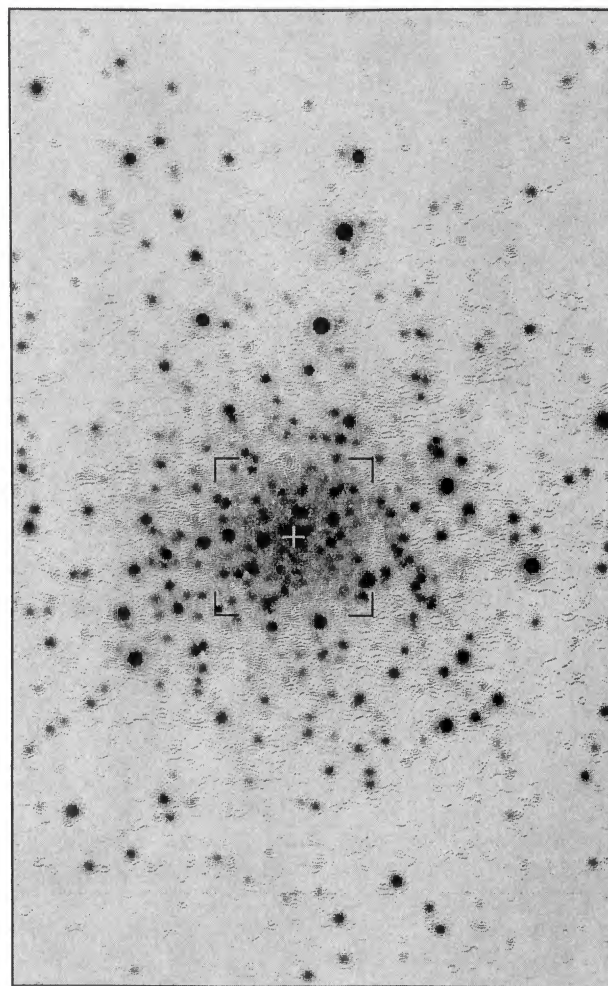
In order to measure color gradients reliably in globular clusters, considerable care must be taken. We will thus briefly describe our technique, which we believe is the best currently available method for this task.

The difficulties in surface photometry of globular clusters derive largely from the bumpy distribution of light in them: most light is contributed by a relatively small number of bright giants, and the  $\sqrt{N}$  fluctuations are the dominant source of errors. More details are given in the reviews by King (1985) and Djorgovski (1987). The difficulties are much more pronounced if one desires to measure colors, and even more so for color gradients.

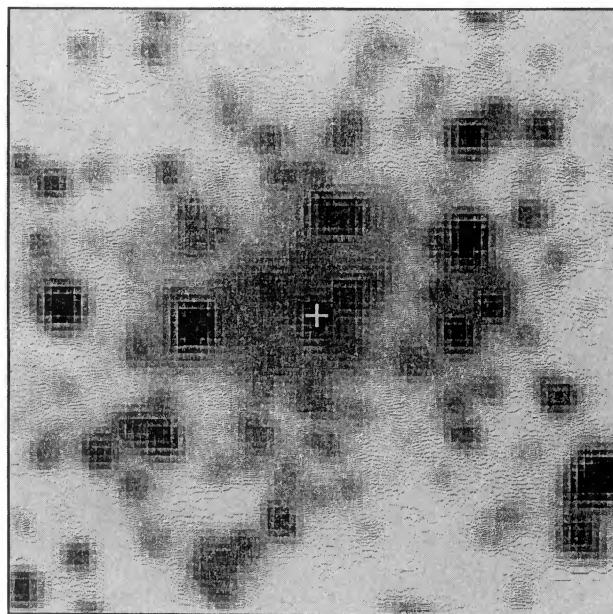
To avoid introducing spurious color gradients through differences in centering, we determined the cluster center carefully on the blue exposure of each set and then used exactly the same center in measuring the images in the other colors. We show in Fig. 1 our adopted center for M30; it is coincident with star 195 of Cordoni and Aurière (1984), and it is also the center of symmetry of the cluster light.

For each frame, we then measured mean surface brightnesses in concentric annuli (radial bins), each divided into eight sectors, using procedures described elsewhere by Djorgovski and King (1986) and Djorgovski (1987). Since the same stars cause the fluctuations in each bandpass, it is extremely important that the boundaries of annuli and sectors used to evaluate the surface brightness correspond exactly in all frames. It is also important that all the frames have the same seeing. We then combined the mean surface brightnesses in each annulus and sector into  $B - V$ ,  $V - R$ , and  $B - R$  colors, and evaluated the mean and the median colors of sectors in each annulus, and the  $\sigma$  of the mean. We believe that this procedure is the optimal way of deriving the photometric color profiles of globular clusters, as it avoids all the pitfalls of aperture photometry (centering, different sampling) and produces realistic internal error bars.

The color profiles are shown in Fig. 2. The symbols represent the mean color, and the error bars the  $\sigma$  of the mean, as derived from the scatter among the eight octants. The centers are redder than their immediate surroundings, because of one or more red giants blended in the central seeing disks. Outside about  $3''$  radius, a color trend is unmistakable. At the center, the two graphs differ, but this is merely a result



(a)



(b)

FIG. 1. (a) Image of the  $R$  band CCD frame obtained at ESO. The field is  $116 \times 186$  arcsec, with north to the top, east to the left. Our assumed cluster center is marked with a cross. (b) A zoom-in on the core region indicated in (a), with the center marked; it coincides with star 195 of Cordoni and Aurière (1984). The zoomed region is 30 arcsec square.



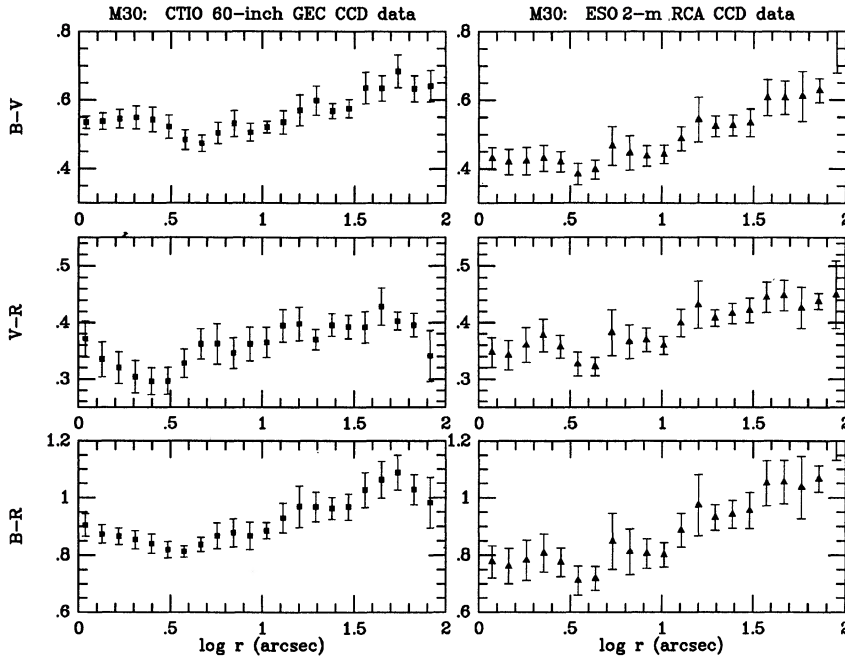


FIG. 2. Color profiles extracted from the CTIO (left) and ESO (right) data frames. The redness in the center is due to an unresolved clump of red giants centered on CA 195. Sky-subtraction errors (*not* included in our error bars) are the cause of the turn-overs at large radii.

of the different seeing in the two datasets. The clump of bright red stars at the center stands out in the ESO images but is smeared out in the poorer-seeing CTIO images. Although the centering was done independently for the two sets of images, it turned out to be the same in both cases. The two sets of color curves have somewhat different zero points, but this merely reflects an incomplete correction for color equation, at the present stage of the much more extensive project that we have under way on this cluster. (The ESO material is close to a standard  $BVR$ , while the CTIO material is still on an instrumental system. The difference has little effect on the gradients.)

We performed a series of least-squares fits to the color gradients in our profiles. The fits are specified by the inner and outer radial limits (3–5 and 50–100 arcsec, respectively), set by the seeing and confusion effects on the inside, and on the outside by the limited size of our CCD frames, which leads to uncertainties in the sky subtraction. We performed the surface photometry as described above, on both the CTIO and ESO frames (“complete”), and in the case of the ESO data also on frames from which the several hundred brightest stars were digitally subtracted (“unresolved background”; cf. Sec. III), and their opposite, frames from which this unresolved background of faint stars was subtracted, leaving only the giants (“background subtracted”). The resulting color-gradient *slopes*, expressed in magnitudes per decade in radius, for three different radial-fitting ranges, are listed in Table I.

The gradients in the original (“complete”) data appear to be highly significant, typically on a  $10\sigma$  level. The colors involving the  $B$  band show more prominent gradients; perhaps that is not too surprising, considering that the red giants dominate the light in both  $V$  and  $R$ . The gradients in the unresolved background are very slight, typically at a  $2\sigma$  level. There is a hint that the gradient in  $V - R$  is reversed after the background subtraction, but again, that is not a very discriminant color baseline.

Peterson (1986) exhibits a similar color gradient in M30, based on photoelectric concentric-aperture photometry. (See also the note to his Table IX for references to earlier studies.) Our integrated color profiles are in good agreement with his compilation, but show much less scatter, as shown in Fig. 3; this is, of course, appropriate for more homogeneous datasets.

TABLE I. Least-squares fits to color-gradient slopes.

Radial range (arcsec) : 3 to 100						
Data set	$B - V$	error	$V - R$	error	$B - R$	error
CTIO Complete	0.121	0.012	0.056	0.011	0.179	0.011
ESO Complete	0.178	0.015	0.094	0.010	0.271	0.023
ESO Unres. bgd.	0.033	0.015	0.104	0.050	0.125	0.057
ESO Bgd. subtr.	0.543	0.050	-0.170	0.110	0.460	0.083
Radial range (arcsec) : 5 to 100						
Data set	$B - V$	error	$V - R$	error	$B - R$	error
CTIO Complete	0.137	0.013	0.036	0.014	0.176	0.020
ESO Complete	0.190	0.022	0.071	0.054	0.274	0.035
ESO Unres. bgd.	0.071	0.021	0.129	0.078	0.141	0.070
ESO Bgd. subtr.	0.591	0.067	-0.249	0.117	0.410	0.093
Radial range (arcsec) : 3 to 50						
Data set	$B - V$	error	$V - R$	error	$B - R$	error
CTIO Complete	0.112	0.015	0.075	0.010	0.185	0.011
ESO Complete	0.172	0.017	0.108	0.011	0.279	0.028
ESO Unres. bgd.	0.024	0.015	0.054	0.043	0.098	0.043
ESO Bgd. subtr.	0.469	0.050	0.053	0.105	0.523	0.120

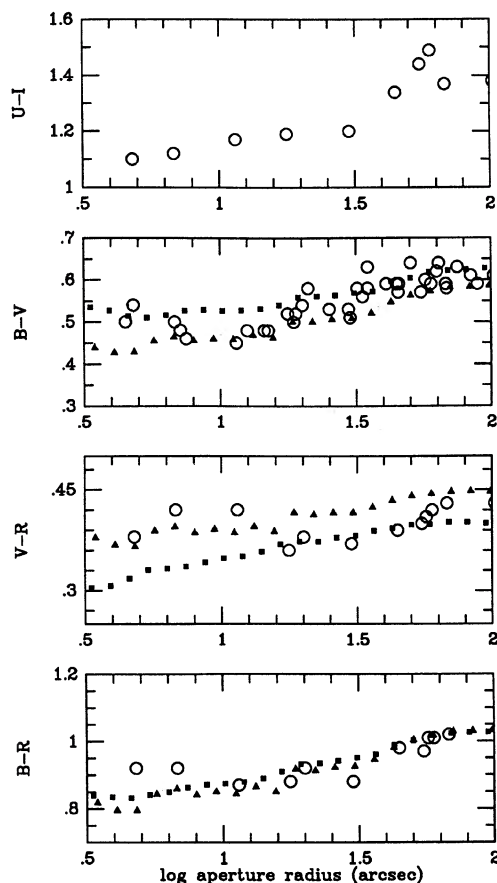


FIG. 3. A comparison of the cumulative color profiles from our data (dark symbols) and the photoelectric measurements collected by Peterson (1986) (light circles). The photometric zero points differ, but the trends are the same. Based on Peterson's ( $U-I$ ) profile, we expect a much stronger effect once CCD data are obtained in these bands and analyzed.

We also performed power law fits to the surface brightness in the central cusp. The best-fit logarithmic slopes are dependent both on the inner cutoff radius  $r_{in}$  (the effect of seeing), and on the outer one  $r_{out}$  as the profiles change eventually into steeper, King-model-like envelopes. Values of  $r_{in} = 1, 1.5$ , or  $2$  arcsec, and  $r_{out} = 10, 15$ , or  $20$  arcsec, were used. As expected, the profile slopes are steeper in the bluer band-passes, differing by a few hundredths from  $B$  to  $R$ . These numbers are not as good a diagnostic as the data presented here, but we mention them because they first called our attention to the color gradient.

The following section describes our attempt to identify the stellar populations responsible for these color gradients.

### III. DISTRIBUTION OF STELLAR TYPES

RST call attention to the indications from the core spectrum of M30 that a substantial part of the light comes from stars of early type, and they discuss various possible contributors. In order to investigate this question, we measured colors and magnitudes of individual stars in the central region, using only the ESO frames, because of the better seeing. These covered a region of  $3.0 \times 1.9$  arcmin, at  $0.37$  arcsec per pixel. The FWHM of star images was  $1.5$  arcsec in  $B$  and  $1.3$  arcsec in  $V$ . The photometry was done with DAOPHOT (Stetson 1987), using a fitting radius of  $2.0$  pixels over most of the frame but  $1.6$  pixels (the smallest that DAOPHOT allows) in the very crowded region within  $10$  arcsec of the center. Color-magnitude arrays for the center and for the outer part of our field are shown in Fig. 4.

Cordoni and Aurière (1984) have also given a color-magnitude array for the central part of M30. They used photographic images with a FWHM similar to ours. Using a different measuring technique, they derived somewhat narrower sequences than ours, but the shape of their red giant branch is distinctly distorted. We have preferred, for homogeneity, to use our own data.

Since M30 has a horizontal branch that is almost exclusively blue (Fig. 4(b)), it is relatively easy to separate the blue-horizontal-branch (BHB) from the red-giant-branch (RGB) stars. What we have done is to divide the color-

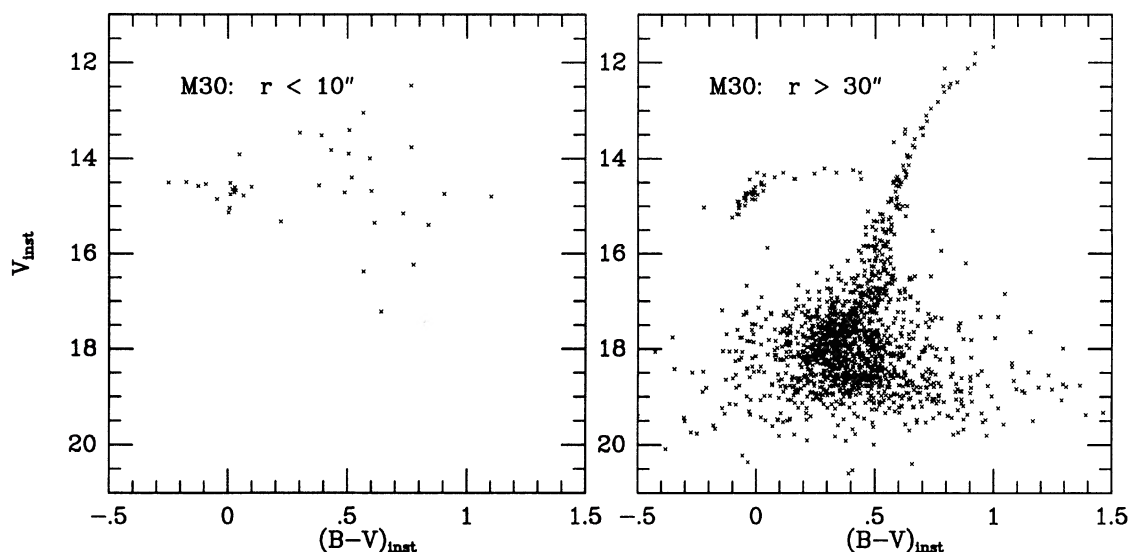


FIG. 4. Color-magnitude arrays of the central part and the outer part of our ESO field.

magnitude array into three parts. For BHB we take stars bluer than  $B - V = 0.4$  and brighter than  $V = 15.5$ , and for RGB the stars in this magnitude range that are redder than  $B - V = 0.4$ . The remaining stars, below  $V = 15.5$ , we call "faint." Finally, by adding up the light of all of these stars in each annulus and subtracting it from the total light in that annulus, we derive a fourth component, which we call "unresolved." (To verify our calculation of this last component, we also used DAOPHOT to subtract all the measured stars from the images and then did surface photometry on the subtracted images; the results were the same.)

The percentage contributions to the blue light at various radial distances are shown in Fig. 5. As expected, there is a general increase in the percentage of unresolved light as we approach the center of the cluster. This is a natural consequence of the increasing difficulty of resolving individual stars. Much, but not all, of the central increase of unresolved light is accounted for by a decrease in the light of the faint group, which are the stars most easily lost to crowding. At the very center, however, there is a further increase in the percentage of unresolved light. It is important to verify that this does not include a component of faint blue stars that is contributing to the blueness of the integrated light—blue stragglers, a faint blue extension of the horizontal branch, or such. For this purpose we show, in Fig. 6, the color profile of the unresolved light from the ESO frames. Except for a turn-up at its outer extremity, which is almost certainly due to

sky-subtraction errors, it is reassuringly bland. The fits presented in Table I suggest a small gradient, but inconclusively. We also show the gradients in the remaining light, i.e., the giants (both BHB and RGB). It is obvious that they must be largely responsible for the observed color gradients.

Figure 5 showed the breakdown of the blue light whose spectrum was observed by RST. It is clear that the BHB stars make a considerable contribution to the light near the center but that the RGB stars become increasingly important farther out. We believe that this BHB light is responsible for the enhanced Balmer lines noted by RST. The lack of blueness in the unresolved light makes it unlikely that there are other important contributors to the Balmer lines in the integrated light.

Strangely, Buonanno *et al.* (1988) noted an apparent radial gradient in the number ratio of BHB and RGB stars, but in the opposite sense. They cover a larger and rather different radial range, however, and they also comment on the perils of small-number statistics. Alcaino and Wamsteker (1982) also noticed an outward increase in  $n(\text{BHB})/n(\text{RGB})$ , but they similarly were comparing our entire region with an exterior region.

#### IV. THE PROBLEM OF THE BHB DISTRIBUTION

Since horizontal-branch stars are generally believed to have a lower mass than red giants, it is surprising to find them more concentrated to the center; this would imply an *inverse* mass segregation, for which we are unable to find a reasonable dynamical explanation.

Before drawing serious conclusions from the difference, however, we should ask how likely it is to be a statistical accident. To examine this question we have compared the radial distribution of BHB and RGB stars, using both a Kolmogorov-Smirnov test and a  $\chi^2$  test. The KS test, of course, requires no binning. We ran it on the two ordered sets and found a probability of 0.152 that two random samplings from the same population would differ this much. Moreover, most of the difference arises in the central region, in which our photometry is less accurate. The cumulative radial distributions are shown in Fig. 7.

For the  $\chi^2$  test it is necessary to separate the data into radial bins. Mindful of the rule of thumb that no bin should contain fewer than five stars, we found it necessary, on account of the dearth of red giants near the center, to take radial steps of at least 20 pixels. However, to avoid having our binning conditioned by the actual distribution of the red giants, we also made another binning with steps of 25 pixels. The counts for both binnings are shown in Table II. For  $\chi^2$  test we used the formula

$$\chi^2 = \frac{1}{MN} \sum_{j=1}^J \frac{(Nm_j - Mn_j)^2}{m_j + n_j}$$

(Breiman 1973), where  $J$  is the number of bins and the samples are  $\{m_j\}$  and  $\{n_j\}$ , of sizes  $M$  and  $N$ . The number of degrees of freedom is  $J - 1$ . This formula gave for the 20 pixel bins a probability of 0.078 that deviations this large would appear in the comparison of two random samplings from the same population; for the 25 pixel bins the probability was 0.38. If we conservatively omit the innermost bin of each set, on account of the larger errors in the central photometry, the probabilities become 0.82 and 0.89, respectively.

It thus appears that the greater apparent concentration of the BHB stars to the center of M30 is not statistically significant.

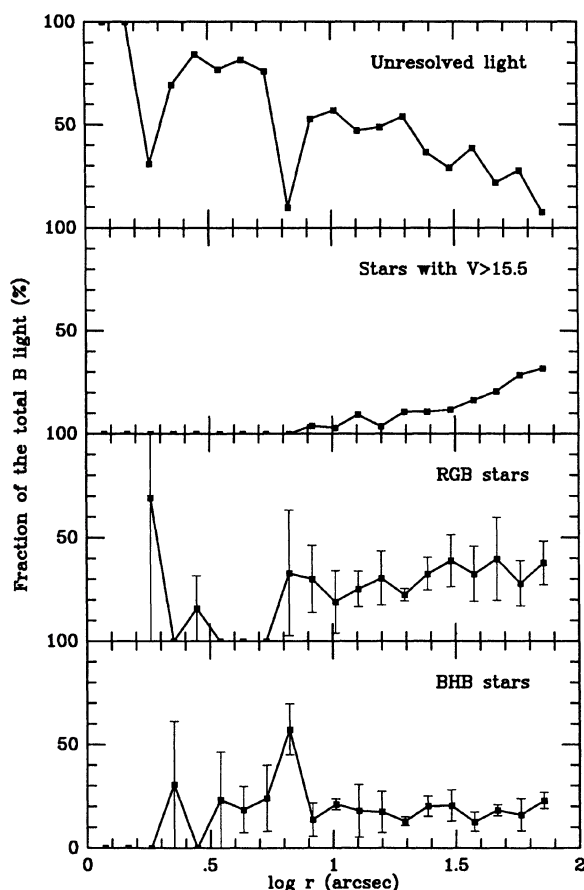


FIG. 5. Percentage contributions of the four components described in the text to the  $B$  light, in the ESO frame.

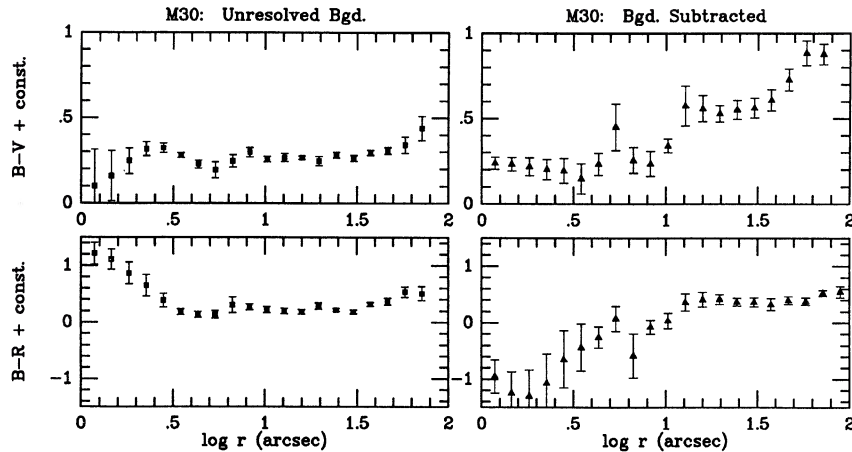


FIG. 6. Color profiles of the unresolved background of fainter stars, from the ESO frames with bright giants subtracted (left), and from their complementary frames, where this unresolved background was subtracted, leaving only the giants (right). The different symbols correspond to two different radial samplings.

cant—particularly in view of the fact that this particular cluster had already been singled out among many for its blue center. However, we note that the  $\chi^2$  test is completely insensitive to the ordering of the data points, and that it could not distinguish between a systematic trend in radius, as observed, and completely erratic deviations from the null hypothesis. In that sense, the KS test is clearly a more appropriate choice but, ideally, one would assume a mathematical form for the effect and use that as a null hypothesis.

On the other hand, the deviations of color-gradient slopes computed in Sec. II from the null hypothesis (constant color) are highly significant, but of course they assume that a linear trend in the log-log plots is indeed a good model for the observed effect. We have examined the contributions to the total light from each of the four groups of stars referred to above, and have tried the effect of artificially adjusting the contributions from the four groups in such a way as to remove the color gradient. We were unable to do so by adjustments whose changes in star numbers were statistically plausible.

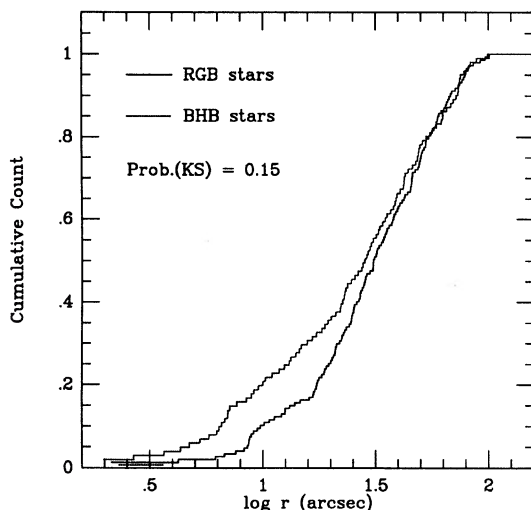


FIG. 7. Cumulative radial distributions of BHB and RGB stars, as measured in the ESO frames.

## V. DISCUSSION AND CONCLUSIONS

We should note that our discussion deals only with the visible light of clusters. Dupree *et al.* (1979) have noted a concentration of ultraviolet light at the centers of several clusters. Since that light may well come from some component other than those whose distribution we have studied here, we cannot take a position on the statistical significance of their results, but it is intriguing that the sense of the effect is the same. Caloi *et al.* (1984) also mention an ultraviolet “spike” in the center of M30.

Prompted by the results of RST, we have also studied NGC 4147, and have obtained similar results in terms of color profiles and number ratios of BHB and RGB stars.

TABLE II. Radial distribution of red-giant-branch and blue-horizontal-branch stars in M30.

r	BHB	RGB	r	BHB	RGB
(pixels)			(pixels)		
0			0		
	15	5		18	12
20			25		
	14	18		15	20
40			50		
	8	21		15	31
60			75		
	15	25		14	22
80			100		
	10	16		11	15
100			125		
	10	12		8	17
120			150		
	11	24		9	17
160			200		
	7	13		11	13
200					
	11	13			
101		147	101		147



Thus we find similar or identical phenomena in both of the two clusters studied so far, but the choice of the two clusters was clearly biased. Our results on NGC 4147 will be presented elsewhere.

To summarize, we have detected color gradients in M30 that appear to be highly significant in terms of the photometric error bars. Our results are in agreement with many previous reports. These gradients appear to be caused by a population gradient in the ratio of BHB and RGB stars. However, the number-ratio gradients do not appear to be significant on the basis of  $\chi^2$  and KS tests.

At this point, however, we three authors are unable to agree on our main conclusion. One of us (I.R.K.) remains skeptical of the significance of any color gradient here. This cluster has already been recognized as extreme, among the hundred clusters that have been examined by aperture photometry, and in a sample as large as this, strange statistical accidents can happen. Peterson's (1986) extensive compilation and discussion shows color gradients in both senses, including a redder envelope in M30 (NGC 7099) and a redder center in NGC 6681, a collapsed-core cluster that has a brightness profile similar to that of M30.

The two other authors (G.P. and S.D.), however, regard the color and population gradients found here as notable and worthy of further study. It is in a logarithmic plot that the gradient is linear, but its monotonic nature is not dependent on this scaling. We are not aware of a quantitative statistical study of any other cluster similar to ours of M30 and NGC 4147. As indicated above, the systematic errors inherent in photoelectric aperture photometry (in particular the differences in centering and relative positioning of apertures) make it difficult to derive reliable color-gradient information from such data, even if the measurements were done in a single session by the same observer. Heterogeneous datasets obtained by different authors and in different conditions are all but hopeless. It is also hard or impossible to evaluate the internal errors. Digital photometry and starcounts (positional color-magnitude-diagram analysis) with a CCD give the only reliable method of studying color and population gradients in a systematic manner, and the data available so

far are scarce. It may be necessary to examine at least a few other clusters in comparable detail, and with an equivalent level of care, before we decide on whether color and population gradients such as those in M30 are a statistical fluke, or evidence of some heretofore unrecognized general phenomenon in the astrophysics of globular clusters.

Bearing this ambiguity in mind, let us offer two very tentative speculations: If the gradients are physically significant, and indeed reflect an inverse mass segregation, it may be worth exploring possible dynamical mechanisms that may produce such situations, even temporarily. This may be plausible in a realistic multimass stellar system, if the collapsed cores oscillate (e.g., McMillan 1986, and references therein). We are not aware of any models detailed enough to test this idea. Alternatively, it is possible that some of our "RGB" stars are actually asymptotic-giant-branch stars (the two types can easily be confused photometrically), which are even lighter than the BHB stars and may be contributing a substantial fraction of the light farther from the center. The observed gradients would then reflect a mass segregation in the normal sense, which may be explainable dynamically without a recourse to some exotic mechanism. The AGB stars should be  $\sim 10\%$ – $20\%$  of the RGB stars by number, but they could be more luminous, on average.

In conclusion, we have measured in M30 a strongly significant color gradient, which seems to be due to a difference in BHB and RGB distributions that is itself *not* statistically significant. Various astronomical conclusions might be reached, depending on the resolution of this paradox. We apologize to the reader for the lack of a clear answer, but none of the three of us entered science because he thought it was easy.

We are indebted to the staffs of ESO and CTIO for their expert help during our observing runs. S. D. wishes to acknowledge partial support from the California Institute of Technology, and G. P. partial support from the Fondazione Ing. A. Gini. This work was also supported in part by contract no. NASA5-28086.

#### REFERENCES

- Alcaino, G., and Wamsteker, W. (1982). *Astron. Astrophys. Suppl.* **50**, 141.
- Bailyn, C. D., Grindlay, J. E., Cohn, H., and Lugger, P. M. (1988a). *Astrophys. J.* **331**, 303.
- Bailyn, C. D., Grindlay, J. E., Cohn, H., and Lugger, P. M. (1988b). In *Dynamics of Dense Stellar Systems*, Proceedings of the CITA Conference, edited by D. Merritt (Cambridge University, Cambridge).
- Breiman, L. (1973). *Statistics* (Houghton-Mifflin, Boston), p. 301.
- Buonanno, R., Caloi, V., Castellani, V., Corsi, C., Ferraro, I., and Piccolo, F. (1988). *Astron. Astrophys.* (in press).
- Buonanno, R., Caloi, V., Castellani, V., Corsi, C., Fusi Pecci, F., and Gratton, R. (1986). *Astron. Astrophys. Suppl.* **66**, 79.
- Caloi, V., Castellani, V., Galluccio, D., and Wamsteker, W. (1984). *Astron. Astrophys.* **138**, 485.
- Chun, M. S., and Freeman, K. C. (1979). *Astrophys. J.* **227**, 93.
- Cordoni, J.-P., and Aurière, M. (1984). *Astron. Astrophys. Suppl.* **58**, 559.
- Djorgovski, S. (1987). In *Globular Cluster Systems in Galaxies*, IAU Symposium No. 126, edited by J. Grindlay and A.G.D. Philip (Reidel, Dordrecht), p. 333.
- Djorgovski, S., and King, I. R. (1986). *Astrophys. J. Lett.* **305**, L61.
- Djorgovski, S., and Penner, H. (1985). In *Dynamics of Star Clusters*, IAU Symposium No. 113, edited by J. Goodman and P. Hut (Reidel, Dordrecht), p. 73.
- Dupree, A. K., Hartmann, L., Black, J. H., Davis, R. J., Matilsky, T. A., Raymond, J. C., and Gursky, H. (1979). *Astrophys. J. Lett.* **230**, L89.
- Friel, E., Heasley, J., and Christian C. (1987). *Publ. Astron. Soc. Pac.* **99**, 1248.
- Hanes, D., and Brodie, J. (1985). *Mon. Not. R. Astron. Soc.* **214**, 491.
- Hesser, J., and Shawl, S. (1985). *Publ. Astron. Soc. Pac.* **97**, 465.
- King, I. R. (1985). In *Dynamics of Star Clusters*, IAU Symposium No. 113, edited by J. Goodman and P. Hut (Reidel, Dordrecht), p. 1.
- Lee, H.-M. (1987). *Astrophys. J.* **319**, 772.
- McMillan, S. L. W. (1986). *Astrophys. J.* **307**, 126.
- Meylan, G. (1982). *Astron. Astrophys.* **110**, 348.
- Nemec, J., and Cohen, J. (1988). *Astrophys. J.* (in press).
- Nemec, J., and Harris, H. (1987). *Astrophys. J.* **316**, 172.
- Peterson, C. J. (1986). *Publ. Astron. Soc. Pac.* **98**, 192.
- Richer, H., and Fahlman, G. G. (1988). In *Dynamics of Dense Stellar Systems*, Proceedings of the CITA Conference, edited by D. Merritt (Cambridge University, Cambridge).
- Rose, J. A., Stetson, P. B., and Tripicco, M. J. (1987). *Astron. J.* **94**, 1202.
- Scaria, K., and Bappu, M. K. V. (1981). *J. Astrophys. Astron.* **2**, 215.
- Stetson, P. B. (1987). *Publ. Astron. Soc. Pac.* **99**, 191.
- Williams, T., and Bahcall, N. (1979). *Astrophys. J.* **232**, 754.
- Zinn, R., and West, M. (1984). *Astrophys. J. Suppl.* **55**, 45.